Phase control and diagnostic of quantum mechanical superposition states

C. Basler, K. Reininger, F. Meinert,* P. N. Ghosh,† and H. Helm‡

Department of Molecular and Optical Physics, University of Freiburg, Stefan-Meier-Strasse 19, 79104 Freiburg, Germany

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We report experiments on the control of the phase η in quantum mechanical superposition states which emerge in electromagnetically induced transparency, $|\psi\rangle = (|1\rangle + e^{i\eta}|2\rangle)/\sqrt{2}$. We interpret our findings in terms of the measurement role that spontaneous emission and the light fields play in selecting the optically dark and bright superpositions. A phase-switching tool is introduced which enables the rapid measurement of the absolute depth of the dark-state minimum and its absolute position on the frequency scale *without* detuning the lasers. The phase-switching technique allows one to determine the relative phase η in an ensemble of quantum mechanical superposition states of a Λ system in a minimally invasive way.

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I. INTRODUCTION

Electromagnetically induced transparency (EIT) is based on the coherent superposition of two ground-state levels $|1\rangle$ and $|2\rangle$ and their interference in the presence of two laser fields ($j = 1,2$) with frequencies ω_j, phases φ_j, and wave number k_j,

$$E_j = E_j^0 \cos[\omega_j t + \varphi_j + k_j z]. \tag{1}$$

The two fields connect the ground states in optical transitions to a common excited level $|3\rangle$ in which case destructive interference of excitation amplitudes in single atoms can occur [1,2]. At two-photon resonance and in the absence of ground-state decoherence, a stationary atom at some spatial position $z = 0$ develops into a coherent superposition, which for equal Rabi frequencies $|g_1| = |g_2| = g$ and at two-photon resonance is

$$|\psi_N\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle - e^{-i(\varphi_1-\varphi_2)}|2\rangle\right). \tag{2}$$

An atom in this noncoupled state is referred to as being dark, it is barred from optical pumping to the excited level. On the other hand, the coupled state

$$|\psi_C\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + e^{-i(\varphi_1-\varphi_2)}|2\rangle\right), \tag{3}$$

often called the bright state, strongly scatters photons.

Aspect and Kaiser [1] point out that a spontaneously emitted photon and the two laser fields take the role of a double Stern-Gerlach filter. The authors use the analogy that spontaneous emission from state $|3\rangle$ of a Λ system is a measurement process equivalent to a horizontal Stern-Gerlach magnet which selects a spin-1/2 particle in state $|\rightarrow\rangle$ or $|\leftarrow\rangle$, corresponding to our ground states $|1\rangle$ and $|2\rangle$. Expressed in terms of the superposition states (2) and (3), the two ground states are

$$|1\rangle = \frac{1}{\sqrt{2}}\left(|\psi_C\rangle + |\psi_N\rangle\right), \tag{4}$$

$$|2\rangle = \frac{1}{\sqrt{2}}\left(|\psi_C\rangle - |\psi_N\rangle\right)e^{+i(\varphi_1-\varphi_2)}. \tag{5}$$

The presence of the two phase-coherent laser fields is equivalent to a second, now vertical Stern-Gerlach magnet which separates atomic states into the projections $|\uparrow\rangle$ and $|\downarrow\rangle$, selecting with equal probability the final states, dark and coupled.

In this manner spontaneous emission in the presence of the two laser fields acts as a two-stage filtering process, leaving the atom with probability 1/2 in either the dark or the bright state. The latter undergoes a new fluorescence cycle and on the next scattering event will again execute this filtering process. From such reasoning one understands that the atom approaches dark-state conditions in an exponential fashion, controlled by the scattering rate [3]

$$\Gamma' = \frac{g^2}{\Gamma} + \gamma, \tag{6}$$

where Γ is the decay rate of the excited state and γ the rate of decoherence of the ground state. An exponentially slow approach to dark-state conditions is expected from such reasoning. This has been confirmed in several experiments [3–6], and a recent quantitative study showed that in the presence of a multilevel Zeeman structure, when spontaneous emission can populate ground states outside the simplified three-level Λ system, the effective rate is reduced below the nominal value of Eq. (6). This study also revealed extreme sensitivity of the dark-state contrast (which is the effective depth of the dark resonance in the regular absorption signal) to noise in the laser phase [3]. In view of the argument of Aspect and Kaiser [1] this is readily understood, because each departure from the fixed phase relationship in the two laser fields requires a renewed two-stage measurement cycle.

The absolute phase of a laser used in an experiment is in general unknown. However, the phase difference of two lasers, $\varphi_0 = \varphi_1 - \varphi_2$, is readily controlled. The central theme of this work concerns the dynamic response of atoms in the dark state to deliberate changes of this laser phase difference which is eventually reflected in the phase of the superposition state (2).

The effect of phase fluctuations on the EIT line shape has attracted attention from theory since at least 1994, when Sultana and Zubairy [7], Fleischhauer *et al.* [8], and Gong *et al.* [9] studied the adverse effect of laser phase fluctuations on the refractive index enhancement and on lasing without population inversion. Sun *et al.* [10] explored the role of

*Institute of Experimental Physics, University of Innsbruck, Austria.

†Department of Physics, University of Calcutta, India.

‡helm@uni-freiburg.de

the relative phase between the excitation paths on the cross phase modulation and two-photon absorption in a four-level double-Λ system. The rigid phase relationship required for EIT in such a system had previously been demonstrated in an experiment by Windholz and co-workers [11].

In 2002 Godone *et al.* [12,13] explored the influence of various modes of phase modulation at EIT resonance on transparency and on the spectrum of coherent microwave emission in a coherent-population-trapping maser. This topic was picked up again in 2007 by Abi-Salloum *et al.* [5] who modeled the enhanced absorption that appeared after a sudden change of laser phase, a feature observed in an experiment by Sautenkov *et al.* [14] in the same year.

On a more general level the phase dependence of an EIT medium provides a means to obtain entanglement between two radiation fields as discussed by Wang *et al.* [15]. The phase difference between the coherent laser fields also plays an important role in the formation of a Bose-Einstein condensate (BEC) pair with defined entanglement and controllable value of the relative phase [16]. Using a dynamic form of EIT, Mair *et al.* [17] demonstrated that the light storage technique developed by Phillips *et al.* [18] is phase coherent. EIT may also be used to transport coherence between two optical channels [19].

Here we report a study of the transient response of EIT to deliberate laser phase changes, both at two-photon resonance and near two-photon resonance. After introducing the experiment and the characteristics of the transient absorption of the EIT medium we discuss aspects of theory which relate to the relative phase in the superposition state and to the dynamic answer of an EIT medium to changes in laser phase. On this basis we provide analytical and numerical results for direct comparison with the experiment. From this analysis we derive a variety of experimental tools which are based on the principle of imposing momentary phase changes on an EIT medium.

II. EXPERIMENTAL SETUP AND TYPICAL RESULTS

We use the experimental setup described recently [3], a buffered Rb cell and two separately tunable lasers which are locked by an optical phase-locked loop (OPLL) [20] to a fixed value $\varphi_0 = \varphi_1 - \varphi_2$. We can change the actual phase difference,

$$\eta(t) = \varphi_0 - \phi(t), \tag{7}$$

of the two copropagating linearly polarized fields prior to merging them in the EIT medium by arbitrary functions $\phi(t)$ by two methods.

One employs an electro-optic modulator (EOM) in the path of one of the laser beams. We use a $LiNbO_3$ phase modulator (30 mm long, Model EO-F10L3-NIR from QBIG). The functional form of $\phi(t)$ is computer controlled, the shortest duration for a phase change by $\phi = \pi$ is estimated to be $\lesssim$200 ns.

In the second method we electronically control the phase using the OPLL which senses two incoming signals. One is formed by mixing the beat frequency of the two lasers, $f_B = (\omega_1 - \omega_2)/2\pi \approx 6.8$ GHz, with the signal from a first frequency generator, $f_1 \approx 7$ GHz. The OPLL compares the electronic difference frequency $f_1 - f_B \approx 200$ MHz with a second reference f_2 and delivers a feedback to the slave laser to equalize $f_2 = f_1 - f_B$. The generators for f_1 and f_2 are phase locked to a master oscillator signal at 10 MHz derived from a GPS receiver. In this fashion the OPLL holds the phase difference φ_0 with a precision of 0.1 rad^2 [20] and the absolute frequency f_B to a precision of 1 Hz. A voltage-controlled phase shifter (Minicircuits, Model JSPHS-150) in the signal line delivering f_2 can be used to shift the phase difference φ_0 by up to $\pm 1.1\pi$ using a d.c. control voltage. This method is rather convenient, but is limited to time scales slower than about 30 μs.

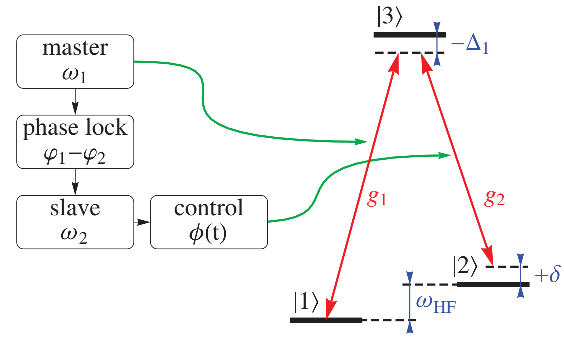

FIG. 1. (Color online) Scheme of experimental frequency and phase control using the simplified Λ system.

Figure 1 explains the experimental frequency and phase control in our Λ system. The master laser is referenced to a nominal detuning $\Delta_1 = 0$ with respect to the $|1\rangle \rightarrow |3\rangle$ transition. The slave laser is operated at a variable detuning $\Delta_1 + \delta$ near the $|2\rangle \rightarrow |3\rangle$ transition. The slave laser is digitally locked in phase to the master laser and the detuning δ is controlled by the OPLL. Prior to entering the Rb cell the phase of the slave laser is shifted by variable functions $\phi(t)$ by using either the EOM or the electronic control discussed above.

All experiments discussed below use the R^- resonance of ^{87}Rb at a field strength of 300 μT. At this magnetic field value this resonance is rather insensitive to magnetic field inhomogeneities, see Fig. 4 in Ref. [3]. This resonance connects the ground states $(F',M_{F'}) = (1,-1)$ and $(2,1)$ with the common excited state $(F,M_F) = (1,0)$. In our experiment we typically monitor the absorption of the sum of *both* laser beams after passage through the 6.5-cm-long glass cell containing natural Rb and neon as buffer gas at a pressure of ≈2.5 kPa. The experimental traces shown typically represent averages over 100 sweeps of the time scale.

A sample result for the transient response of absorption of the EIT medium to phase switching at two-photon resonance is given in Fig. 2. The sequence of events in this experiment is as follows. Dark-state conditions had been established at times $t < 0$ by keeping a fixed laser phase difference φ_0. At $t = 0.3$ ms the two-photon detuning is suddenly switched by $\delta/2\pi = \Delta f = +5$ kHz, a value larger than the full width at half maximum of the dark-state resonance, $\Gamma'/\pi \approx 1400$ Hz. The trace up to 2 ms shows the transient response to exiting the dark-state condition. These are Rabi oscillations at the frequency δ. The oscillations are damped into the regular thermal absorption signal at the rate Γ', as discussed in Ref. [3]. At time $t = 2.2$ ms the frequency is switched back to two-photon

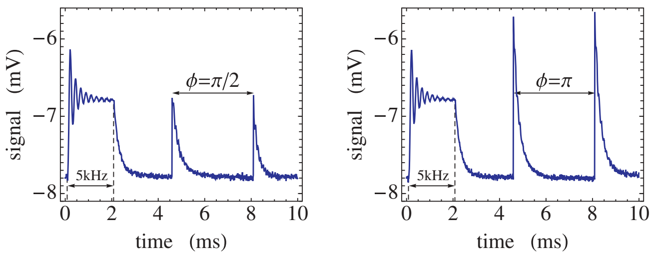

FIG. 2. (Color online) Transient response of absorption to phase switching with the EOM by $\phi = \pm\pi/2$ and by $\pm\pi$ when the lasers are on two-photon resonance. A common photodiode signal of -0.3 V was subtracted. Such a subtraction is done for all experimental photodiode signals shown in this paper in order to avoid excessive digits in the y label. For referencing the absorption scale a frequency jump of $\Delta f = 5$ kHz is implemented in the time window from $0.3 < t < 2.2$ ms.

resonance and the well-known exponential decay into the dark state ensues. We use this frequency-shifting procedure to provide a calibration for the depth of the EIT window.

The actual phase-switching experiment begins at time $t = 4.7$ ms when the phase of laser 2 is changed by the respective value of ϕ. At time $t = 8.2$ ms the phase of laser 2 is changed back to $\phi = 0$ again. We observe that the three exponential decays, beginning at 2.2, 4.7, and 8.2 ms, closely match, signifying the two-stage measurement process discussed above. We also note that the transient response to phase switching leads to peaks of equal height, within experimental fluctuations, when switching by ϕ and back to zero again.

We introduce the dimensionless quantity brightness b as a measure of the peak in the transient response which occurs immediately after switching the phase of the laser. We reference the brightness measure to the difference between the absorption in the dark state and the thermal absorption, which is away from two-photon resonance. In the examples shown in Fig. 2 the brightness is $b^{\pm} \approx 1$ and $b^{\pm} \approx 2$ for switching by $\phi = \pm\pi/2$ and $\phi = \pm\pi$, respectively.

We observe that the responses $b^{\pm}$ are unequal when the two-photon detuning is not zero. Results for phase switching at detunings $\delta/2\pi = \Delta f = \pm 400$ Hz, near two-photon resonance, are shown in Fig. 3. In this case the dynamic response to opposing phase changes is not identical, but mirrors the sign of two-photon detuning. A simplistic explanation is that a phase change corresponds to a momentary change of frequency and the two opposing phase changes correspond to moving closer to and farther from two-photon resonance. In order to quantify the switching phenomena that appear in Figs. 2 and 3 we next introduce some general aspects from EIT theory.

III. THEORY

In the basis of the bare states $|1\rangle$, $|2\rangle$, and $|3\rangle$ shown in Fig. 1 and using real valued Rabi frequencies we write for the Hamiltonian ($\hbar = 1$)

$$\mathcal{H} = \frac{1}{2}\begin{pmatrix} 2\Delta_1 & 0 & g_1\, e^{i\varphi_1} \\ 0 & 2(\Delta_1+\delta) & g_2\, e^{i\varphi_2} \\ g_1\, e^{-i\varphi_1} & g_2\, e^{-i\varphi_2} & 0 \end{pmatrix}. \tag{8}$$

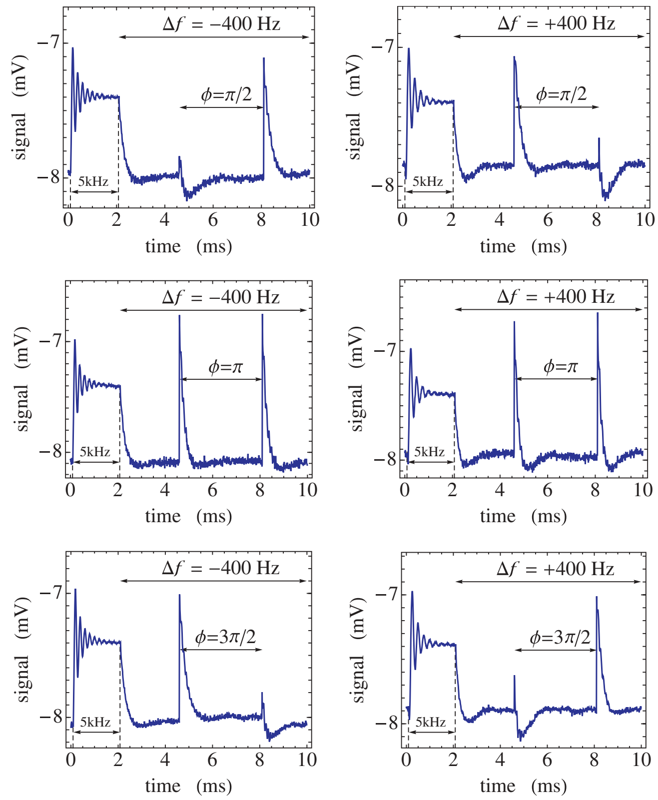

FIG. 3. (Color online) Transient response of absorption to phase switching by $\pm\pi/2, \pm\pi$ and by $\pm 3\pi/2$ at two-photon detunings of $\Delta f = \pm 400$ Hz.

A second complete orthogonal set is given by the family of states $\mathcal{F} = \{|3\rangle, |\Psi_C\rangle, |\Psi_N\rangle\}$, where we have

$$|\Psi_N\rangle = \frac{1}{\sqrt{g_1^2+g_2^2}}(g_2|1\rangle - g_1 e^{-i\varphi_0}|2\rangle), \tag{9}$$

$$|\Psi_C\rangle = \frac{1}{\sqrt{g_1^2+g_2^2}}(g_1|1\rangle + g_2 e^{-i\varphi_0}|2\rangle). \tag{10}$$

The coupling elements among the $\mathcal{F}$ states are

$$\langle 3|\mathcal{H}|\Psi_N\rangle = 0, \tag{11}$$

$$\langle 3|\mathcal{H}|\Psi_C\rangle = \frac{e^{-i\varphi_0}}{2}\sqrt{g_1^2+g_2^2} = \frac{\sqrt{2}\, g\, e^{-i\varphi_0}}{2}, \tag{12}$$

$$\langle \Psi_C|\mathcal{H}|\Psi_N\rangle = -\delta\,\frac{g_1\, g_2}{g_1^2+g_2^2} = -\frac{\delta}{2}. \tag{13}$$

The rightmost expressions in Eqs. (12) and (13) stand for equal Rabi frequencies. The interpretation of these equations is as follows. For a given phase relation between the lasers there is a fixed phase between the two ground states in which the dark state is formed, $\eta = \varphi_0$. This dark state develops according to the actual value of φ_0, provided φ_0 is constant over the characteristic time $1/\Gamma'$ required for forming the dark superposition. The coupling element (11) shows that atoms in the dark state are barred from scattering photons from either laser field. Atoms in the bright state scatter at a rate twice the value for the incoherent atom (12). When the two-photon detuning is not zero, the bright and dark state show the coupling (13).

We note that if both laser fields are turned off concurrently, the atomic superposition state that has formed continues to exist. The atom has stored the relative laser phase relation as the phase φ_0 and the minus sign in the superposition (9). If at any time later the lasers are turned on again, stationary atoms will still be dark, provided no dephasing occurred in either the atom or the two laser fields.

A. Sudden phase change at two-photon resonance

If the laser phase difference is suddenly switched from φ_0 to $\varphi_0 + \phi$, atoms which were prepared in the dark state at phase φ_0 instantly find themselves in a superposition of the new states

$$|\hat{\Psi}_N\rangle = \frac{1}{\sqrt{g_1^2 + g_2^2}}\,(g_2|1\rangle - g_1 e^{-i(\varphi_0-\phi)}|2\rangle), \tag{14}$$

$$|\hat{\Psi}_C\rangle = \frac{1}{\sqrt{g_1^2 + g_2^2}}\,(g_1|1\rangle + g_2 e^{-i(\varphi_0-\phi)}|2\rangle), \tag{15}$$

and scatter photons corresponding to the fractional $|\hat{\Psi}_C\rangle$ character. At two-photon resonance and immediately after switching the laser phase by ϕ the previously dark atom can be described by the superposition

$$|\phi\rangle = \frac{g_2^2 + g_1^2 e^{-i\phi}}{g_1^2 + g_2^2}\,|\hat{\Psi}_N\rangle + \frac{(1 - e^{-i\phi})g_1 g_2}{g_1^2 + g_2^2}|\hat{\Psi}_C\rangle. \tag{16}$$

The fractional coupled state character $\langle\hat{\Psi}_C|\phi\rangle$ gives rise to scattering by the coherent atomic sample at the rate

$$g_{\rm coh}^2 \propto |\langle\hat{\Psi}_C|\phi\rangle\langle 3|\mathcal{H}|\Psi_C\rangle|^2 = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2}\sin^2\left(\frac{\phi}{2}\right). \tag{17}$$

Our brightness measure relates this rate to the rate of scattering outside the EIT resonance by the incoherent sample

$$g_{\rm inc}^2 \propto \frac{g_1^2 g_2^2}{2(g_1^2 + g_2^2)}, \tag{18}$$

leaving us with the simple expression for brightness

$$b^{\pm} = g_{\rm coh}^2/g_{\rm inc}^2 = 2\sin^2(\pm\phi/2). \tag{19}$$

We confirmed this sinusoidal dependence in many experiments, see for example Fig. 4.

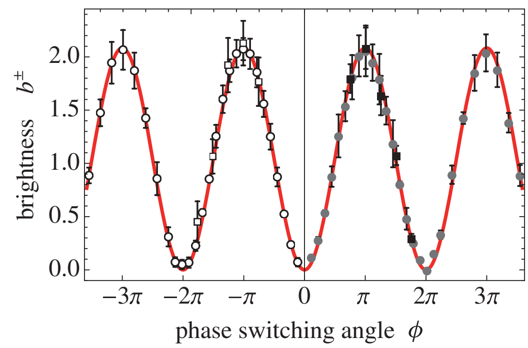

FIG. 4. (Color online) The dimensionless quantity brightness immediately following a phase change at two-photon resonance. The open and full markers refer to measurement of b^- and b^+, respectively. Squares and circles refer to different days of measurement. The fitted $\sin^2$ function has a prefactor of 2.08 ± 0.02, quite in line with the amplitude predicted by Eq. (19).

B. Master equation

To simulate the response of the EIT medium to sudden phase changes when two-photon resonance is not obeyed or when using an experimental ramp for changing the phase over a specified duration and form, we must resort to solutions of the quantum master equation

$$\frac{d}{dt}\rho(t) = -i\,[H,\rho(t)] + \mathcal{L}\rho(t). \tag{20}$$

Here $\rho(t)$ is the interaction picture density matrix which describes a Λ system driven by two phase-controlled lasers. The total Hamiltonian is

$$H = H_0 + H_{\rm int}. \tag{21}$$

H_0 describes the electronic degree of freedom of the three-level atom of Fig. 1 with the detuning of our laser fields from the unperturbed electronic energy differences ω_{3j},

$$\Delta_1 = \omega_1 - \omega_{31}, \quad \Delta_2 = \omega_2 - \omega_{32}. \tag{22}$$

To account for the experimental phase control $\phi(t)$ we separate out an overall phase φ_1 and describe the two co-propagating lasers with the interaction Hamiltonian

$$H_{\rm int} = \frac{g_1}{2}e^{i[k_1x(t)-\vec{k}_1\cdot\vec{v}\,t]}|3\rangle\langle 1| + \frac{g_2}{2}e^{i[k_2x(t)-\vec{k}_2\cdot\vec{v}\,t-\eta(t)]}|3\rangle\langle 2| + \text{H.c.} \tag{23}$$

Here $x(t)$ denotes the atom position which moves with velocity $\vec{v}$. A thermal Rb atom in a buffered cell travels only a fraction of a mm in a microsecond. As the wave numbers for Rb are $k_1 \approx k_2 = k = 8 \times 10^6\ \text{m}^{-1}$ and $k_1 - k_2 \approx 1.4\ \text{cm}^{-1}$ and our experimental dark-state coherence times are shorter than about 30 ms, we may neglect the spatial and Doppler terms in Eq. (23). This leads to the interaction Hamiltonian

$$H_{\rm int} = \frac{g_1}{2}|3\rangle\langle 1| + \frac{g_2}{2}e^{-i\eta(t)}|3\rangle\langle 2| + \text{H.c.} \tag{24}$$

and the dressed state Hamiltonian

$$H = \frac{1}{2}\begin{pmatrix} 2\Delta_1 & 0 & g_1 \\ 0 & 2\Delta_2 & g_2 e^{-i\,\eta(t)} \\ g_1 & g_2 e^{+i\,\eta(t)} & 0 \end{pmatrix} \tag{25}$$

in which the time-dependent term $\eta(t)$ of Eq. (7) appears. The time dependence of this term is slow on the scale of laser frequencies ($2\pi \times 380$ THz) and the laser frequency difference ($\approx 2\pi \times 6.8$ GHz), but not necessarily slow with respect to temporal changes of the density matrix elements. The latter occur at rates proportional to Γ', typically in the kHz range.

For the Liouvillian $\mathcal{L}$ in Eq. (20) we consider the rate of spontaneous decay of level $|3\rangle$ into the ground states $|1\rangle$ and

$|2\rangle$ with rates Γ_1 and Γ_2 where $\Gamma = \Gamma_1 + \Gamma_2$ and a rate γ for dephasing of the ground-state coherence,

$$\mathcal{L}\rho(t) = \begin{pmatrix} \Gamma_1\,\rho_{33}(t) & -\gamma\,\rho_{12}(t) & -\frac{\Gamma}{2}\,\rho_{13}(t) \\ -\gamma\,\rho_{21}(t) & \Gamma_2\,\rho_{33}(t) & -\frac{\Gamma}{2}\,\rho_{23}(t) \\ -\frac{\Gamma}{2}\,\rho_{31}(t) & -\frac{\Gamma}{2}\,\rho_{32}(t) & -\Gamma\,\rho_{33}(t) \end{pmatrix}. \quad (26)$$

Taking $\Gamma_1 = \Gamma_2 = \Gamma/2$ we obtain from Eq. (20) the coupled equations

$$\rho'_{11} = \tfrac{1}{2}[\,\Gamma\rho_{33} + i\,g_1\,(\rho_{13} - \rho_{31})\,], \quad (27)$$

$$\rho'_{22} = \tfrac{1}{2}[\Gamma\rho_{33} + i\,g_2 e^{i\,\eta}\,(\rho_{23} - \rho_{32})], \quad (28)$$

$$\rho'_{12} = \frac{i}{2}[-g_1\rho_{32} + g_2 e^{i\eta}\rho_{13}] - [\gamma + i(\Delta_1 - \Delta_2)]\rho_{12}, \quad (29)$$

$$\rho'_{31} = \frac{-i}{2}[g_1\,(\rho_{11} - \rho_{33}) + g_2\,e^{-i\eta}\rho_{21}] - (\Gamma - 2i\,\Delta_1)\rho_{31}, \quad (30)$$

$$\rho'_{23} = \frac{i}{2}[g_1\,\rho_{21} + g_2 e^{-i\eta}(\rho_{22} - \rho_{33})] - \left(\frac{\Gamma}{2} + i\,\Delta_2\right)\rho_{23}, \quad (31)$$

$$\rho'_{33} = \frac{-i}{2}[g_1\,(\rho_{13} - \rho_{31}) + g_2\,e^{i\,\eta}(\rho_{23} - \rho_{32})] - \Gamma\rho_{33}. \quad (32)$$

For simplicity of writing we omitted the explicit time dependence of the matrix elements and of η.

The observed absorption coefficient is related to the imaginary part of the susceptibilities,

$$\chi_j(t) = \frac{2n_{\text{Rb}}}{\epsilon_0\hbar}\frac{|\vec{d}_{j3}|^2}{g_j}\,\rho_{j3}(t), \quad (33)$$

where n_{Rb} is the density of Rb atoms and d_{j3} is the respective transition dipole moment.

C. Analytical solution for sudden phase change

To consider sudden phase jumps when being off-resonance we consider as the initial system the steady state near two-photon resonance ($\delta < \Gamma'$) for equal Rabi frequencies $g_1 = g_2 = g$ and $\Gamma \gg g$. Vanier *et al.* [21] have shown that under these conditions the optical coherences in Eqs. (30) and (31) follow adiabatically the rf coherence in Eq. (29). This is because their response time to a perturbation (of the order of Γ) is much faster than that of the rf coherence (of the order of g^2/Γ). Setting $\rho'_{31} = \rho'_{23} = 0$ and solving Eqs. (30) and (31) with the initial populations $\rho_{33} = 0$ and $\rho_{11} = \rho_{22} = 1/2$ gives ρ_{31} and ρ_{23} in terms of ρ_{21}. Inserting these results into Eq. (29) we obtain an analytical solution $\rho_{21}(t)$ which we use in Eq. (30) to obtain an analytical solution for $\rho_{31}(t)$. We do not write out the complicated expression explicitly because a simple practical result appears for the measure brightness. We have defined brightness as the difference in absorption just before and immediately after switching the phase at $t = 0$, relative to the difference in absorption just before switching the phase and the absorption far away from resonance,

$$b(\phi,\delta) = \frac{\text{Im}[\chi_{31}(\text{t} < 0,\phi = 0,\delta)] - \text{Im}[\chi_{31}(\text{t} = 0,\phi,\delta)]}{\text{Im}[\chi_{31}(\text{t} < 0,\phi = 0,\delta)] - \text{Im}[\chi_{31}(\text{t} < 0,\phi = 0,\delta \gg \Gamma')]}.$$

In the limit $\Gamma > \delta$ and $g^2/\Gamma \gg \gamma$ we obtain

$$b(\phi,\delta) = 2\,\sin^2\left(\frac{\phi}{2}\right) + \frac{\Gamma\,\delta}{g^2 - \delta^2}\,\sin\phi. \quad (34)$$

At resonance this expression reduces to the brightness measure derived from the wave-function picture, Eq. (19).

D. Numerical solutions

To simulate the entire transient for both frequency and phase shifting such as shown in the right column in Fig. 5 we numerically solve the quantum master equation under the conditions $\text{Tr}[\rho] = 1$ and $\rho'_{11}(t) + \rho'_{22}(t) + \rho'_{33}(t) = 0$. As switching function for the phase we employ $\phi(t) = \phi\,\sin^2[\frac{\pi}{2\tau}(t - t_s)]$ in the range $(t_s \leqslant t \leqslant t_s + \tau)$. Here τ is the duration over which the phase changes and $\phi(t < t_s) = 0$ and $\phi(t \geqslant t_s + \tau) = \phi$. Three-level simulation of the realistic Rb system requires us to account for the pressure-broadened width of the excited state, $\Gamma \approx 2\pi \times 190$ MHz, see Sec. IV B of Ref. [3]. Simulations are performed at the time resolution of 1 μs and then integrated to resemble the experimental sampling resolution of 10 μs/step of the photodiode.

IV. COMPARISON OF THEORY AND EXPERIMENT

We discussed in the context of Fig. 2 that the brightness after phase switching by ϕ follows a simple sinusoidal pattern according to Eq. (19) and this response appears independent of the sign of the phase change. The transient response is rather different when the lasers are away from two-photon resonance. The brightness responses $b^{\pm}$ are then unequal as seen in the examples in Fig. 3. To explore the sign dependence further we show in the left column of Fig. 5 transients at varying values of the detuning δ. In each case the phase change is $\pm\pi/2$. As for the data in Fig. 3 where the results for positive and negative detuning by 400 Hz are compared, the experimental data given in the left column of Fig. 5 show an asymmetry when changing the sign of detuning. The higher transient peak appears when the sign of the phase change $d\phi/dt$ is equal to the sign of detuning δ when the overall phase change ϕ is in the range $\{0,\pi\}$, but opposite when the phase change is between $\{\pi,2\pi\}$.

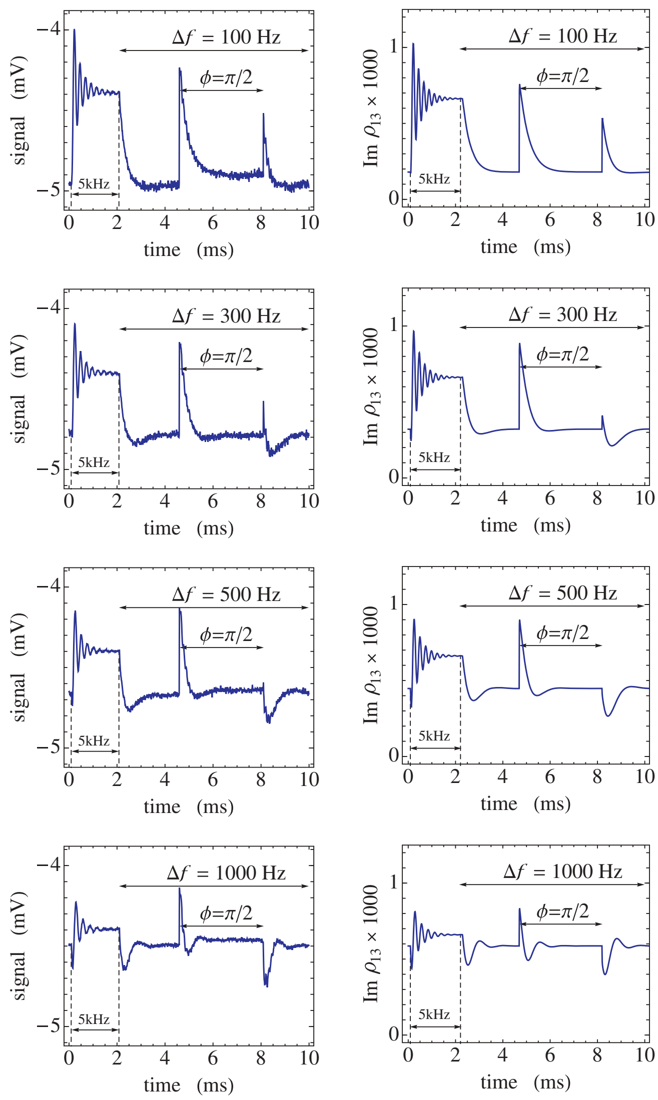

FIG. 5. (Color online) Experimental transients for phase shifting by $\frac{\pi}{2}$ are shown in the left column for various values of the detuning. For referencing the absorption scale, a frequency jump by $\Delta f = 5$ kHz is implemented in the time window from $0.3 < t < 2.2$ ms. The simulation in the right column uses the parameters $g_1 = 2\pi \times 210$ kHz, $g_2 = -2\pi \times 420$ kHz, and $\gamma = 2\pi \times 20$ Hz. These Rabi frequencies correspond to intensities which are 10% higher than using Eq. (35) in [3] with the experimental intensities $I_1 = 210$ and $I_2 = 155\ \mu\text{W/cm}^2$. In view of the uncertainty of laser beam profiles this is considered as quite consistent.

A second feature in the transient response to a phase change appears on a longer time scale. This is the dip in absorption. The depth of the dip increases with the magnitude of detuning. A simplistic explanation is that a phase change mimics to some extent a frequency change. When the frequency change is in the direction of the EIT minimum, the absorption passes through the region of decreased absorption hence the dip in absorption is deeper. We note, however, that this dip occurs well after the phase change has been completed.

These experimental details are fully confirmed by numerical simulations of the time-dependent quantum master equation. The right column in Fig. 5 shows the result of such simulations carried out at the appropriate experimental parameters using a single Λ system. There is *quantitative* agreement in the complex transient shapes. The absolute scaling of the photodiode response signal is an empirically adjusted parameter, but it is identical for all simulated data shown in the right column in Fig. 5.

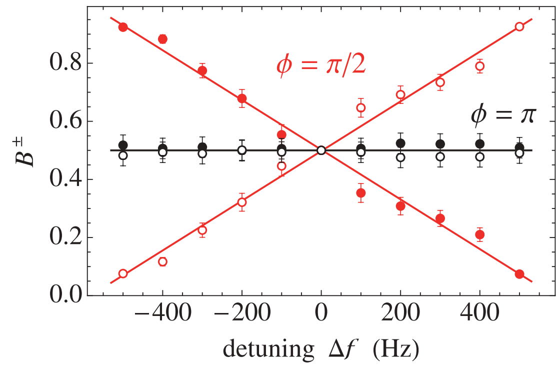

FIG. 6. (Color online) The dimensionless brightness ratio $B^{\pm} = b^{\pm}/(b^{+} + b^{-})$ as a function of detuning for two values of the phase shift. Open circles refer to B^{+}, full circles refer to B^{-}. The full lines represent Eq. (35), see text.

We explored experimentally the sensitivity of the transient peak for phase shifting to unequal and varying laser intensities at detunings $\delta \lesssim \Gamma'$ (data not shown). We found that the ratio of the two peak heights, b^{+} and b^{-}, which appear immediately after switching the phase by $+\phi$ and by $-\phi$ is fairly insensitive to the laser intensity ratio. This finding has greater significance since it allows us to define the brightness ratios $B^{\pm} = b^{\pm}/(b^{+} + b^{-})$ as the measure for the magnitude of detuning from EIT resonance.

The ratios $B^{\pm}$ reflect the fundamental features of the superposition state in relation to laser field parameters. The ratios depend sensitively on the sign and magnitude of the two-photon detuning δ and on the sign and magnitude of the difference between the laser-field phases and the phase of the superposition state. Using the analytical result in Eq. (34) we derive the relationship

$$B^{\pm} = \frac{b^{\pm}}{b^{+} + b^{-}} = \frac{1}{2} \pm \frac{\delta}{2\Gamma'} \cot(\phi/2) \tag{35}$$

$$= \frac{1}{2} \pm \frac{\delta}{2\Gamma'} \quad \text{for} \quad \phi = \frac{\pi}{2}, \tag{36}$$

and $B^{\pm} = \frac{1}{2}$ for $\phi = \pi$. We plot experimentally determined ratios $B^{\pm}$ versus the detuning δ in Fig. 6. We note that they follow rather precisely the dependence predicted by relation (35). The full lines are the results of a least-squares fit using the function $B^{\pm} = B_0 \pm \frac{\delta}{2\Gamma'}$. The values obtained for B_0 are in the range 0.5 ± 0.002.

The slopes obtained for phase switching by $\phi = \pm\pi/2$ in Fig. 6 are $\frac{1}{2\Gamma'} = 8.6/(2\pi) \pm 0.7 \times 10^{-4}\ \text{Hz}^{-1}$. This yields $\Gamma' = 3650 \pm 280$ Hz from which we deduce a FWHM of the EIT resonance of $\Gamma'/\pi = 1160 \pm 90$ Hz on the regular frequency scale.

V. DISCUSSION

The quantitative consistency of experiment and model predictions shows us that phase switching can be applied to accomplish otherwise difficult and time-consuming diagnostic measurements. One diagnostic tool is the detection of the magnitude and sign of two-photon detuning from *absolute* EIT resonance. This information is accessible from the asymmetry of the phase-switching response, that is, from the magnitude and sign of either B^+ or B^-. The highest sensitivity for such a measurement is the choice of $\phi = \pm\pi/2$.

A second application is to use the height of the transient peak as a tool for measuring the absolute depth (the contrast) of an EIT minimum. It is most gratifying that both measurements can be accomplished *without* the requirement of detuning the frequency of the lasers, an intrinsically slow procedure, see Fig. 10 of Ref. [3]. We note that our phase-switching method is related to but different from the periodic phase modulation technique employed in the coherent-population-trapping maser [12] for derivation of a frequency error signal. These novel experimental tools deserve further attention because the *instant* response to phase switching, i.e., the measures $b^\pm$, can be monitored in a minimally invasive way as we discuss next.

The obvious reason for a spiked transient response is that immediately after switching the phase the superposition state is viewed with a laser phase *inappropriate* for the steady-state conditions established before switching the phase. This view can be recorded with only minute disturbance as we show in Fig. 7. Here the phase shift $\phi = \pi$ is accomplished over a time window $\tau = 200$ ns and set back to zero again after $\Delta t = 10$ and 100 μs. The initial spike in absorption is identical for both durations, but for the longer duration a substantial reformation of the phase of the superposition occurs. This is apparent from the exponential decay following the phase-switching pulse for $\Delta t = 100$ μs. Obviously the degree of reformation is controlled by the laser intensities used in this experiment.

On the other hand we see at $\Delta t = 10$ μs that the dark-state base line remains practically unchanged. Our current experimental setup is limited in that the shortest useful duration, Δt, is limited by the time of acquisition of the photodiode signal sampling, $\approx$10 μs.

Obviously the spiked transient response can be measured by employing a burst of short pulses of both lasers. Pulses of duration of $\approx$1 μs would suffice provided the photodiode signal recording can be kept at this rate. Due to the slow response of the medium, such short flashes of laser light at incorrect phase angle will not significantly alter the medium and can therefore be applied repeatedly. Indeed if a burst of such pulses is provided with varying phase shift ϕ, the ensuing spiked response of brightness responds with a nearly nondestructive measurement of the relative phase.

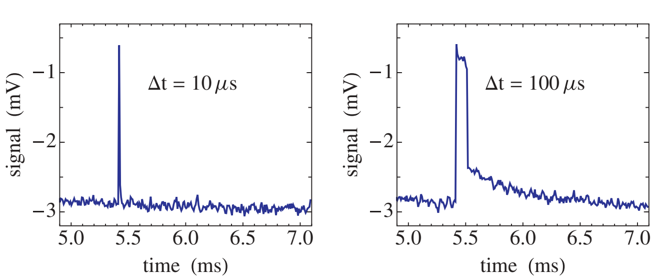

FIG. 7. (Color online) Response of transient absorption to a sudden phase change by $\phi = \pi$ over a time duration of Δt at two-photon resonance.

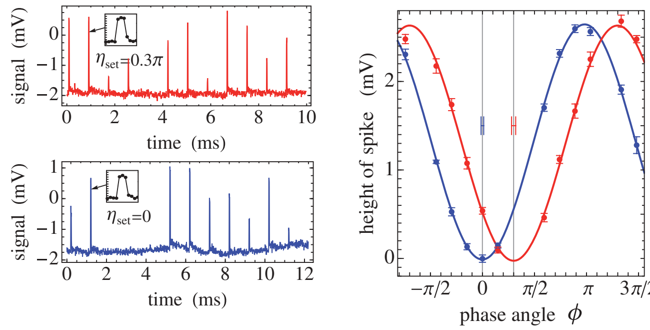

FIG. 8. (Color online) Two measurements of the phase of a superposition state prepared at $\eta_{set} = 0.3\pi$ and $\eta_{set} = 0$ using a burst of 12 short pulses of differing phase shift ϕ. The traces at the left give the spiked response of the two samples in transmission. The curves shown on the right are least-squares fits of the function $a + b\sin^2(\phi - \eta_{fit})$ to the data. The fitted phases agree well with the phase at which the state was prepared.

To demonstrate the proof of concept, we show in Fig. 8 the result of two such burst measurements. In each case we first prepared an EIT sample at a relative phase angle η_{set} (with φ_0 at nominally zero) by permanently setting the electro-optic phase control to the chosen value, once to $\eta_{set} = 54^\circ = 0.3\pi$ and once to $\eta_{set} = 0$. In each case we kept the EIT beams on all the time but then changed the phase in an identical sequence of 12 pulses (pulse duration of $\Delta t = 30$ μs) of differing phase shift ϕ (randomly chosen). In the example shown the phase shifts were implemented using EOM voltages of {400, -200, 0, -40, 40, 240, 280, 160, 360, -120, 200, -80} V. Fitting the height of the spiked response to Eq. (17) using the function $a + b\sin^2(\phi - \eta_{fit})$ delivers the phase measures $\eta_{fit} = 53.3 \pm 2.6^\circ$ and $\eta_{fit} = -0.1 \pm 1.5^\circ$. The errors signify two standard deviations. This method may be employed in ensembles of quantum mechanical superposition states to derive the relative phase in the superposition state.

VI. CONCLUSION

We show that manipulation of the relative laser phase difference in an EIT configuration with Λ-type Rb atoms enables controlled exit from and entry into the dark state and sudden switching of the EIT medium from dark to bright and reverse. Various applications of phase switching are tested and compared to theoretical models. We find that a single rapid switch of the phase of one laser by π gives rise to a characteristic transient response of the refractive index which reveals the absolute depth of the EIT minimum *without* the need for detuning lasers. We also find that the characteristic response to two consecutive phase changes of $+\pi/2$ and $-\pi/2$ provides an experimental measure for the magnitude of detuning and the direction of detuning from EIT resonance, an obvious advantage in many experimental situations. Phase information of laser radiation is lost in all measurements of intensity by a detector. However, an EIT state may be prepared to store information on the phase of a laser relative to a reference laser field. Using prudent phase switching this information can be recovered later in a minimally invasive way. This enables the determination of the relative phase in an ensemble or condensate of quantum mechanical superposition states for which the relative phase between the two ground-state vectors is unknown.

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