Terahertz pulse propagation in the near field and the far field

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Received June 14, 1999; accepted September 10, 1999; revised manuscript received September 14, 1999

We present a detailed investigation of the propagation properties of beams of ultrashort terahertz (THz) pulses emitted from large-aperture (LA) antennas. The large area of the emitter is demonstrated to have substantial influence on the temporal pulse profile in both the near field and the far field. We perform a numerical analysis based on scalar and vectorial broadband diffraction theory and are able to distinguish between near-field and far-field contributions to the total THz signal. We find that the THz beam from a LA antenna propagates like a Gaussian beam and that the temporal profile of the THz pulse, measured in the near field, contains information about the temporal and spatial field distribution on the emitter surface, which is intrinsically connected to the carrier dynamics of the antenna substrate. As a result of pulse reshaping, focusing of the THz beam leads to a reduced relative pulse momentum, with implications in THz field-ionization experiments.

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1. INTRODUCTION

For the past decade a common method to generate subpicosecond, single-cycle electromagnetic pulses (terahertz or THz pulses) for spectroscopic purposes (THz time-domain spectroscopy) has been to excite a biased semiconductor or an electro-optic crystal with a femtosecond light pulse. In the case of a biased semiconductor, the acceleration of photogenerated carriers results in emission of the THz pulse, and in the case of an electro-optic crystal, optical rectification of the excitation pulse generates the THz pulse.

The description and understanding of the propagation of THz pulses in optical systems has recently attracted considerable research effort. Uhd Jepsen and colleagues measured and calculated the distribution of the THz field in front of a lens-coupled point-source dipole emitter. Bromage et al. made thorough investigations of the spatiotemporal shaping of THz pulses by diffraction through conductive apertures. You and Bucksbaum calculated the propagation characteristics of unipolar THz pulses through optical systems, and Feng et al. calculated properties of the temporal profile and spectrum of a THz pulse in the vicinity of a focus. Winnewisser et al. measured and discussed spectral and temporal pulse shaping by frequency-selective surfaces. Budiarto et al. modeled and measured the temporal profile of a THz pulse in the near field of a large-aperture (LA) antenna.

One reason for the interest in the propagation characteristics of THz pulses is that the temporal shape of the THz pulse can be measured with great accuracy on a femtosecond time scale. Therefore numerical models of ultrashort-pulse propagation can be stringently tested. Furthermore, results obtained with single-cycle THz pulses can be scaled in frequency to predict the behavior of single-cycle pulses in the visible, which have not yet been experimentally realized.

Pulses emitted from a biased, LA antenna tend to show unipolar E-field amplitude and consequently possess a transverse momentum; they are therefore of particular importance in field ionization experiments, pioneered by the groups of Bucksbaum, Jones, and Noordan (see, for instance, Refs. 9 and 10 and references therein). The apparent unipolar pulse shape is the result of the generation process (a steplike transient current in the emitter), combined with the effects of propagation into the far field (differentiation of the temporal pulse shape). Because of the unipolar shape of the THz pulse, a LA antenna is the most commonly used emitter type for field ionization experiments in spite of the relatively long duration of the pulse. In Fig. 1 we compare the pulse shape of a THz pulse generated by a transient photocurrent in a LA antenna with the shape of a THz pulse generated by optical rectification on an unbiased GaAs surface. The ringing seen after the pulse generated by optical rectification is caused in part by residual water vapor absorption and in part by a strong phonon resonance at 5.3 THz in the ZnTe detector. The two pulses are normalized to their peak amplitudes in Fig. 1. With this normalization the area of the unipolar pulse is at least 50 times larger than that of the pulse generated by optical rectification. The two pulses were measured under similar conditions, the only difference being the generation mechanism.

In the work presented here we perform experimental and numerical investigations of the propagation of THz pulses from a biased, LA antenna with dimensions larger than the characteristic wavelength of the radiation. Because of the general nature of our numerical calculations, the main conclusions drawn here are also valid for other types of LA antennas, e.g., unbiased semiconductor surfaces and electro-optic crystals.

The paper is organized as follows. First, we outline theoretical results from scalar and vectorial diffraction
theory and present numerical simulations of THz pulse propagation. Then we present experimental measurements of the frequency-dependent spatial distribution (the radiation pattern) of the electric field emitted from a LA antenna and compare the experimental radiation patterns with results from diffraction theory. Complementary to the experimental results in the frequency domain, we then present measurements of the temporal profile of the THz pulses as the detector is moved from the near field into the far field of the emitter. Again, the experimental data are compared with results from diffraction theory.

2. THEORETICAL BACKGROUND

In this section we give an overview of the theoretical background of the description of the generation and propagation of THz pulses emitted from a LA antenna. We will use the words antenna and emitter interchangeably. The calculation of the temporal and spectral radiation pattern from a THz emitter is a diffraction problem. A phase-sensitive integration of the electric field distribution on the emitter surface determines the electric field in front of the emitter. The distribution of the electric field on the emitter surface is, in turn, determined by carrier dynamics in the semiconductor substrate of the emitter and by the geometry of the emitter.

All calculations presented here are performed with parameters identical to those of the emitters used in our experiments. The emitter has a gap length \( l = 10 \) mm and height \( h = 10 \) mm. The emitter is excited by a pump beam with a \( 1/e \) radius of 3.5 mm. We use a Cartesian coordinate system with the \( x \) axis along the direction of the applied bias field and the \( z \) axis along the propagation direction of the THz beam. The origin of the coordinate system is taken as the center of the LA emitter.

A. Scalar and Vectorial Diffraction Theory

Owing to the extremely large spectral contents of a typical THz pulse, it is necessary to employ broadband versions of the standard diffraction integrals to give a full description of the radiation pattern from a THz emitter. It is useful to study the radiation pattern both in the frequency domain and in the time domain. By doing so, it is possible to see how individual frequency components propagate and how the temporal profile of the THz pulse is modified during propagation.

The calculation of the radiation pattern in front of the THz emitter can be performed by the application of Huygen's principle of superposition of spherical wavelets. If a monochromatic wave is emitted from a plane surface \( A \) (the THz emitter surface) the scalar representation of the electric field in the space in front of the emitter can be written as\(^\text{11}\)

\[
U(x, \nu) = \frac{\nu}{ic} \int_{A} U(x') \frac{\exp(i2\pi \nu r'/c)}{r'} \cos(n, r') dx',
\]

where \( x, n, x', \) and \( \nu \) are the observation point, the inward normal of the emitter surface, an integration point on the emitter surface, and the frequency, respectively. \( r' = x' - x \) and \( r' = |r'| \) are the vector and distance from observation point to integration point, respectively. The cosine factor is known as the inclination factor. Equation (1) is also known as the Rayleigh–Sommerfeld diffraction integral. The general principle of the integration is simple: An ensemble of spherical wavelets \( \exp(i2\pi \nu r'/c)/r' \) with amplitudes \( U(x') \) are emitted from the antenna surface \( A \) and adds up in amplitude and phase to form the electric field at the position \( x \).

The time-domain version of the Rayleigh–Sommerfeld diffraction integral, also known as the broadband Huygens–Fresnel diffraction integral, is obtained by inverse Fourier transformation of Eq. (1):\(^\text{11}\)

\[
u(x, t) = \frac{\cos(n, r')}{{2\pi}cr'} \frac{du(x', t - r'/c)}{dt} dx'. \tag{2}
\]

This integral shows that in the far field, the scalar representation of the field at a point \( x \) is proportional to the time derivative of the surface field on the THz emitter at the retarded time \( t - r'/c \). The two integrals (1) and (2) both represent a scalar approximation of the full electric field vector.

In the near field of the emitter the electric field cannot be treated as a scalar field owing to the large curvature of the wave front. Diffraction integrals for the full electric field vector exist, and for a monochromatic wave the electric field vector \( \mathbf{E}(x) \) is given by\(^\text{12}\)

\[
\mathbf{E}(x, \nu) = \frac{1}{2\pi} \mathbf{\nabla} \times \int_{A} \frac{[n \times \mathbf{E}(x')] \exp(i2\pi \nu r')}{r'} dx'. \tag{3}
\]

This integral is also known as the Smythe–Kirchhoff diffraction integral. The inverse Fourier transformation of the electric field is defined as

\[
\mathbf{E}(x, t) = \int_{-\infty}^{\infty} \mathbf{E}(x, \nu) \exp(-i2\pi \nu t) d\nu. \tag{4}
\]

To calculate the time-dependent spatial distribution of the electric field \( \mathbf{E}(x) \), the frequency-domain expression of the field in Eq. (3) is applied to the inverse Fourier transformation, Eq. (4):
\[ \mathbf{E}(\mathbf{x}, t) = \int \frac{1}{2\pi} \left[ \nabla \times \left( \int_{A} [\mathbf{n} \times \mathbf{E}(\mathbf{x}', \nu)] \right) \times \exp(i2\pi \nu r'/c) \right] \exp(-i2\pi \nu t) d\nu \]

\[ = \nabla \times \int_{-2\pi}^{2\pi} \frac{1}{2\pi} \left[ \mathbf{n} \times \int_{A} \mathbf{E}(\mathbf{x}', \nu) \right] \times \exp(i2\pi \nu r'/c) \exp(-i2\pi \nu t) d\nu \]

\[ = \nabla \times \int_{A} \frac{1}{2\pi r'} \left[ \mathbf{n} \times \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{x}', t - r'/c) \right] \times \exp(-i2\pi \nu t - i r'/c) d\nu \]  \hspace{1cm} (7)

In comparison with Eq. (4) we see that the term in curly brackets in Eq. (7) is the Fourier transformation of \( \mathbf{E}(\mathbf{x}', t - r'/c) \). This leads to the following result:

\[ \mathbf{E}(\mathbf{x}, t) = \nabla \times \int_{A} \frac{1}{2\pi r'} [\mathbf{n} \times \mathbf{E}(\mathbf{x}', t - r'/c)] d\nu. \]  \hspace{1cm} (8)

Hence the electric field vector at an arbitrary point in space can be found if the time-dependent surface field of the THz emitter is known.

In the far-field limit, results from scalar [Eq. (2)] and vectorial [Eq. (8)] diffraction theory merge to the same expression. The scalar field function \( u \) is, in this situation, identical to the \( x \) component of the electric field vector \( \mathbf{E} \). Near the propagation axis in the far field the \( y \) component of the electric field vanishes, indicating that the THz beam can be treated as a plane wave in that region.

All four diffraction integrals presented here are valid for \( r' \gg \lambda \), which at 1 THz is \( r' \gg 0.3 \) mm. All numerical simulations and experiments are performed for \( z > 5 \) mm, where the theory is expected to be valid.

**B. Surface Field of the Emitter**

The time-dependent electric field at the surface of the emitter is needed in the diffraction integrals described in Subsection 2.A. A widely accepted model describing the behavior of the surface field is the current surge model. Within this model, the THz field on the LA emitter is given by:

\[ \widetilde{E}_{THz}(t) = -E_0 \frac{\sigma_s(t) \eta_0}{\sigma_s(t) \eta_0 + 1 + \sqrt{\epsilon}}. \]  \hspace{1cm} (9)

The surface conductivity induced by an optical beam of intensity \( I_{opt}(t) \) is defined as

\[ \sigma_s(t) = \frac{e(1 - R)}{h\nu} \int_{-\infty}^{t} \mu(t - t') I_{opt}(t') \times \exp(-t - t')/\tau_{car} \, dt', \]  \hspace{1cm} (10)

which is a convolution of the time-dependent mobility \( \mu \) and the carrier decay. \( R \) is the reflection coefficient of the optical pump beam, and the factor \( e/h\nu \) implies that each photon of the optical beam generates an electron in the conduction band.

The reflection coefficient \( R \) is a function of the refractive index and the absorption coefficient of the substrate. Therefore the time dependence of \( R \) should in principle be included in Eq. (10). However, we find in a control experiment that the change of the reflection coefficient \( R \) is due to excitation of the emitter is small enough to be ignored.

The reason for the small change in reflectivity is that the absorption of excitation photons is saturated, owing to a limited amount of carriers that can be excited from the valence band. With a high photon density incident on the emitter, the absorption depth is therefore larger in comparison with the absorption depth at low photon densities. This effect reduces the amount of free carriers at the emitter surface and hence reduces the change in reflectivity. Our experiments show that at excitation levels comparable with the other experiments in this paper, the reflectivity increases ~5% at the time of excitation, with a slow, exponential decay back to the normal reflectivity. The time constant of this decay is several hundred picoseconds. Therefore, within the accuracy of our model, we find that the time dependence of the reflectivity can be neglected.

The time-dependent mobility can be expressed as

\[ \mu(t) = \mu_{dc} - (\mu_{dc} - \mu_i) \exp(-\Gamma t), \]  \hspace{1cm} (11)

where \( \Gamma \) is the inverse carrier relaxation time of the initially hot carriers, \( \mu_{dc} \) is the steady-state mobility, and \( \mu_i \) is the mobility of carriers immediately after excitation. Nuss et al.\(^4\) measured the time-dependent mobility of carriers in GaAs following excitation by a 625-nm 100-fs laser pulse and found that the mobility increases with a time constant of several picoseconds from a low initial value to the much higher stationary value, as the photoexcited carriers relax to the minimum of the conduction band.

The temporal shape of the surface field, calculated from Eq. (9), is a steplike function, by which the steepness of the step is determined by the rise time of the optical excitation and the rise time of the mobility. In the case of excitation at 800 nm with a 100-fs pulse, the finite mobility rise time will be the limiting factor. We expect the mobility to increase faster than was found in Ref. 14, since we excite carriers to a lower initial energy in the conduction band of GaAs (120 meV for 800 nm as opposed to 550 meV for 625 nm excitation wavelength). The carriers relax toward the conduction band minimum mainly by electron–phonon scattering processes. Kash et al.\(^5\) measured the electron–phonon scattering time to be 165 fs. The longitudinal-optical phonon energy in GaAs is 36 meV, so in our case we expect a relaxation time of the order of 500–600 fs.

The decay of the electric field follows the decay of the carrier density, which in intrinsic, semi-insulating GaAs is slow (several hundred picoseconds) on the time scale considered here. In Fig. 2 we have plotted the temporal behavior of the emitter surface field [Eq. (9)] for different choices of parameters. To generate the curves in Fig. 2 we used the following standard values: \( \Gamma = 2 \) THz, \( \eta_0 = 377 \) O, \( \epsilon = 12.25 \), \( \tau_{laser} = 0.1 \) ps, \( \tau_{car} = 600 \) ps, \( \mu_{dc} = \)
5800 cm²/Vs, and a constant reflectivity $R = 0.3$. In each panel of the figure, the solid curve corresponds to this parameter set. In Fig. 2(a) we vary the mobility $\mu_{dc}$ of the carriers. The lower the static mobility is, the slower the rise time of the surface field is and the lower the final field strength is. In Fig. 2(b) the effect of varying the inverse carrier relaxation time $\Gamma$ is illustrated. As expected, we see that a slow relaxation time results in a slow onset of the surface field. In Fig. 2(c) we vary the carrier recombination time $t_{car}$ and find, not surprisingly, that with a fast recombination time, the surface field disappears on a fast time scale. The nonexponential decay seen in this situation is caused by the saturation properties of Eq. (9).

The electric field from the bias applied across the emitter gap is not uniform. In the planar geometry of the emitter the field is enhanced near the electrodes, which one can see by solving Poisson's equation for the given geometry. The electric field distribution is further modified if the contact between the electrodes and the emitter substrate is nonohmic. This results in a depletion zone, and hence a strong field enhancement, near the electrodes. In the numerical modeling presented in Subsection 2.C we apply a phenomenological field distribution of the form
\[
E_B(x) = E_c + (E_e - E_c)(x/l)^n, \tag{12}
\]
where $E_c$ and $E_e$ is the electric field at the center of the emitter area and at the electrodes, respectively. The exponent $n$ determines how fast the field drops from the value at the electrodes to the value at the center.

C. Results from Gaussian Propagation Theory

The integration of the Rayleigh–Sommerfeld diffraction integral [Eq. (1)] shows that if the spatial intensity profile of the exciting laser beam is Gaussian, the THz far field $U(x, y, z)$ accurately reproduces the spatial profile of a Gaussian beam with beam waist $w_0$ given by the spot size of the pump beam located at the emitter surface. If the pump beam is blocked by the electrodes, the THz beam is diffracted by the resulting aperture. On the basis of these observations we can describe the propagation of the THz pulse in the far-field region by normal Gaussian beam propagation theory.

If the THz beam propagates without focusing, the spot size of the field distribution (e⁻¹ radius) at a distance $L$ from the emitter is
\[
w_{\text{unfocused}}(\nu) = \frac{cL}{\nu \pi w_0} \left[1 + (\nu \pi w_0^2/cL)^2\right]^{1/2}. \tag{13}
\]
If a lens with focal length $f$ is inserted in the THz beam at a distance $f$ from the emitter ($f - f$ geometry), a focus is formed at a distance $2f$ from the emitter with a spot size of
\[
w_{\text{focused}}(\nu) = \frac{fc}{\nu \pi w_0}. \tag{14}
\]
Focusing the THz beam clearly modifies the spatial distribution of the frequency components. Hence on the propagation axis the spectral and temporal shape of the THz pulse is modified by focusing. For a given pulse energy the on-axis field strength of a Gaussian beam is inversely proportional to the spot size. Therefore, at a distance $L = 2f$ from the emitter, the ratio of the focused field strength to the unfocused field strength is given by
\[
\frac{E_{\text{focused}}}{E_{\text{unfocused}}} = 2\left[1 + (\nu \pi w_0^2/cf)^2\right]^{1/2}. \tag{15}
\]
All frequency components are concentrated on a smaller area by focusing, which leads to a higher total field strength at the focal point.

In Fig. 3 the frequency-dependent spot size of a THz beam unfocused and a THz beam focused in a $f - f$ geometry, determined by Eqs. (13) and (14), and the field enhancement factor given by Eq. (15) are shown. The initial spot size on the THz emitter, which is the only free parameter, is adjusted to mimic the experiments presented later in the paper, $w_0 = 3.4$ mm. The unfocused

Fig. 2. Model electric field on emitter surface for different parameter choices: effect of varying (a) the static mobility, (b) the carrier relaxation rate, and (c) the carrier recombination time.
spot size (long-dashed curve in Fig. 3) shows a strong divergence at low frequencies. At high frequencies the spot size approaches the initial spot size, as expected from ray optics. The focused spot size (short-dashed curve in Fig. 3) shows a less pronounced divergence at low frequencies. At high frequencies the field ratio (solid curve in Fig. 3) is linear with respect to frequency, i.e., \[ \frac{E_{\text{focused}}}{E_{\text{unfocused}}} \] \( \propto f \). This is identical to a differentiation in the time domain. Hence in the high-frequency limit the temporal shape of a pulse at the focus of a lens is the time derivative of the unfocused pulse. In the low-frequency limit the field ratio approaches a constant value, resulting in an unmodified pulse shape. The borderline between the two regions is at \[ \nu = \frac{2cf}{\pi w_0^2} \], which, under our experimental conditions (\( w_0 = 3.4 \) mm, \( f = 120 \) mm), is at \( \sim 2 \) THz. The bandwidth of the THz pulses emitted from biased, LA antennas typically extends to as much as 1.5 to 2 THz, so the shape of the pulse is not expected to be modified dramatically. A pulse with higher bandwidth, e.g., emitted by a \( x^2 \) process, will be more strongly modified by focusing.

In Fig. 4 the beam propagation, based on Gaussian ABCD formalism,\(^16\) is shown for the two situations outlined here. The 1/e diameter of the beam is plotted for different frequency components, starting at 0.1 THz (largest divergence), in steps of 0.2 to 1.9 THz (smallest divergence). The initial spot size is 3.4 mm in accordance with our experimental conditions. In Fig. 4(a) the beam is propagating freely; in Fig. 4(b) the beam is focused by a \( f = 120 \) mm lens in a 2f configuration.

### 3. NUMERICAL SIMULATIONS OF THz PULSE PROPAGATION

We numerically integrated the vectorial broadband diffraction integral (8) to visualize the temporal evolution of the radiation pattern from a 10 mm \( \times \) 10 mm LA emitter. If the emitter is excited by a plane wave front, only two of the three THz field components have nonvanishing values, that is, the component parallel to the emitter bias field \( (x) \) and the component parallel to the propagation axis \( (z) \). In Fig. 5 the \( x \) component (perpendicular to the propagation axis) of the electric field distribution is shown for [Fig. 5(a)] \( t = 333 \) ps (wave front at \( z = 99.8 \) mm) and [Fig. 5(c)] \( t = 32 \) ps (wave front at \( z = 9.6 \) mm) after excitation of the emitter. A color version of Fig. 5 can be found at our homepage\(^17\), together with an animation sequence illustrating the pulse propagation. The \( z \) component (parallel to the propagation axis) of the field distribution at the same times is shown as Figs. 5(b) and 5(d). In these plots the transition from the near field to the far field can be seen. As a result of the large emitting area, retardation effects are important in the near field. This leads to a highly structured pulse of several picoseconds duration. The structure of the pulse shape reflects the spatial distribution of the electric field on the emitter surface. The effect of field enhancement near the electrodes can be seen as sharp, triangular boundaries. In the far field, retardation of the electrical field is much less important, and the THz field has evolved into a single, short pulse. This reflects the fact that in the far field, the temporal shape of the THz pulse is proportional to the time derivative of the electric field on the surface of the emitter. The density plot of the \( z \) component at \( t = 32 \) ps after excitation [Fig. 5(d)] has been scaled by a factor of 2.5, and the density plot showing the \( z \) component at \( t = 333 \) ps [Fig. 5(b)] has been scaled by a factor of 17; i.e., the \( z \) component of the electric field at larger distances is very small near the propagation axis. Exactly on the propagation axis \( (x = 0 \) mm) the \( z \) component vanishes; it increases in magnitude with increasing distance from the propagation axis. We see a sign change at \( x = 0 \) mm [in Figs. 5(b) and 5(d), positive field values are shaded in white, and negative values are shaded in black], indicative of the curved wave front of the THz beam.
4. EXPERIMENTAL SETUP

To verify the numerical simulations presented in Section 3, we have performed experiments to measure the spatiotemporal field distribution in the near field and in the far field of a LA emitter. The near-field setup is schematically illustrated in Fig. 6(a); Fig. 6(b) shows the setup for determination of the far-field distribution. The laser source used in the experiment is a regeneratively amplified Ti:sapphire system, delivering 1-mJ pulses of 80-fs duration and 800-nm wavelength at 1-kHz repetition rate. Part of the beam is used to generate the THz radiation. A LA GaAs antenna with 1-cm electrode spacing is biased by 5–8-kV dc voltage and excited by a 50-μJ laser pulse (the pump pulse). The electrodes are attached to the emitter by a conducting mixture of epoxy glue and silver paint. The transient surface conductivity set up by the pump pulse results in the emission of an intense electromagnetic pulse with a duration of ~0.5 ps and a peak field strength of several kV/cm.

The THz pulses are detected by free-space electro-optic sampling. Briefly, the time-dependent electric field of the THz pulse is used to retard the phase of an initially linearly polarized femtosecond probe pulse, derived from

Fig. 5. (a and b) Density plots of the spatiotemporal distribution of the x and z components of the THz field 333 ps after excitation; (c and d) the x and z components of the THz field 33 ps after excitation.

Fig. 6. (a) Experimental setup for near-field measurements; (b) setup for far-field measurements. GM, guiding mirror (plane or off-axis paraboloidal); NBS, nonpolarizing beam splitter; PBS, polarizing beam splitter; PD, photodiode; EO, electro-optic crystal; +HV, high-voltage supply.
the pump pulse on a beam splitter, which is propagating collinear with the THz pulse inside an electro-optic crystal, in this case ZnTe. The duration of the probe pulse is shorter than the duration of the THz pulse; therefore the temporal shape of the THz pulse can be measured by changing the relative delay between the THz pulse and the probe pulse while monitoring the phase retardation of the probe beam. The phase retardation is measured as the difference in light intensity of the two polarization components of the probe beam with use of a balanced photodiode setup, as illustrated in Fig. 6. If the balanced detector setup is optically biased to the quarter-wave point, the instantaneous electric field of the THz pulse is proportional to the measured phase retardation. We send the probe pulse into the ZnTe detector crystal from the back surface with respect to the direction of the incoming THz pulse. The advantage of this geometry is that no pellicle beam splitter is needed in front of the detector. Hence we can measure the shape of the THz pulse in the extreme near field of the emitter.

A. Frequency-Resolved Spatial Radiation Patterns
In this section we describe a measurement of the spatial radiation pattern emitted from a biased, LA emitter of 10 mm × 10 mm, and the emitter is biased by a 5-kV static potential drop across the photoconductive gap. The laser beam excising the emitter is matched to the size of the gap, so that the electrodes are not significantly illuminated. The distance from emitter to detector is kept fixed at 240 mm, and we measure the radiation pattern in two situations. First, a plane mirror with a diameter of 80 mm is used to guide the THz beam to the emitter (unfocused propagation), and next an f = 120-mm off-axis paraboloidal mirror with a diameter of 70 mm is used as guiding optics in a 2f configuration. This configuration focuses the THz beam to a diffraction-limited spot at the detector. The spatial resolution of the detector is determined by the spot size of the sampling beam, which in our experiment is ~0.5 mm. In Fig. 7 temporal traces of the THz pulse at various positions of the detector perpendicular to the propagation axis of the THz beam, in the direction of the THz field polarization, are shown for [Fig. 7(a)] the unfocused beam and for [Fig. 7(b)] the focused beam. The effect of focusing is clear. In the unfocused beam there is only little change in the shape and amplitude of the THz pulse over several millimeters near the propagation axis, whereas the shape of the pulse in the focused beam is severely disturbed when the detector moves away from the propagation axis. In Fig. 8 we have performed a Fourier transformation of the individual traces in Fig. 7 to look at the spatial distribution of the frequency components of the THz pulse. Data for the unfocused beam are shown in Fig. 8(a) and for the focused beam in Fig. 8(b). Again, the effect of focusing is obvious. In the unfocused beam the spectral distribution of the pulse is insensitive to the exact position of the detector, and the 10% bandwidth limit (indicated by small vertical bars) varies gently and less than 50% over the full range of the detector. In the focused beam the bandwidth variation is more substantial, with a clear peak close to the propagation axis and a rapid decrease at off-axis positions. We observe an apparent enhancement of the bandwidth on the propagation axis, in comparison with the unfocused beam. Pulses in the focused beam are shorter than pulses in the unfocused beam, FWHM 0.6 ps versus 0.8 ps. These observations are explained by the focusing process, since in the present geometry, high-frequency components are focused more tightly than low-frequency components. Hence we expect to observe a larger bandwidth in a focused beam on the propagation axis. The apparent reduction of spectral content at large off-axis position is an indication of the well-known fact that the spot size of a focused beam is smaller than the spot size of the unfocused beam.

If we make cross sections at specific frequencies of the data in Fig. 8, we obtain frequency-dependent radiation patterns of the beam. We show radiation patterns of the unfocused beam in Fig. 9(a) and of the focused beam in Fig. 9(b). The radiation pattern is shown at frequencies from 0.15 to 1.2 THz, in steps of 0.15 THz. Together with the experimental results we show fits, assuming a Gaussian profile of the beam at each frequency component. We use a simple Gaussian beam profile, since in the far field the integration of diffraction integrals (1) and (3) in the far field basically results in a Gaussian beam profile. To verify that the THz beam propagates like a Gaussian beam, in Fig. 10 we have plotted the frequency dependence of the experimental spot size w(r) and the spot size of an ideal Gaussian beam, obtained from Eqs. (13) and
The calculated curves in Fig. 10 require as the only free fitting parameter the initial spot size $w_0$ at the emitter. The fits in Fig. 10 were obtained with $w_0 = 3.4 \text{ mm}$, in perfect agreement with the experimental conditions. The unfocused beam size is too large by a factor of 1.2 to 1.5 in the range from 0.5 to 1.0 THz. We believe that this is caused by diffraction on a post mounted in the vicinity of the THz beam. The diffraction signal can be seen in Fig. 7 as a small oscillation starting at the $t = 3.5 \text{ ps}$ trace and moving to shorter times at larger off-axis positions. In the case of the focused beam the spot size is reduced; therefore the THz beam is unaffected by the post, resulting in a better fit. We observe that the measured spot sizes at low frequencies are too small compared with the calculated spot sizes, both in the unfocused and in the focused beams. We attribute this discrepancy to the finite size of the guiding optics in the THz beam path. The plane mirror in the experiment has a radius of 40 mm, and the off-axis paraboloidal mirror has a radius of 35 mm, which agrees reasonably well with the observed spot sizes at low frequencies.

If the pulse area at the center of the THz beam $A = \int E(t)/E_{\text{peak}} \, dt$, normalized to the peak value of the electric field, is compared with the focused and unfocused beams, we find that the unfocused pulse has a normalized area of 0.38 ps, compared with a normalized area of 0.22 ps of the focused pulse. This, together with the modified pulse duration, is the direct manifestation of the increased content of high-frequency components of the pulse in the focused beam. The decrease of the normalized pulse momentum on focusing is caused by an enhanced, negative feature following the main peak. This feature is the first indication of the onset of bipolarity of the pulse. We do not observe a complete transition from a unipolar pulse in the unfocused beam to a symmetric bipolar pulse in the focused beam. To observe this experimentally, either a pulse with larger bandwidth or a focusing element with shorter focal length must be used.

**B. Spatially Resolved Temporal Pulse Profiles**

We have measured the temporal shape of the THz pulse in the near field and in the far field of the THz emitter. The pulse shape measured in the near field contains information about the field distribution on the emitter surface, as will be shown below. In Fig. 11 we show the THz pulse shape recorded at $z = 16 \text{ mm}$ from the emitter, at different off-axis positions. The pulse shape in general consists of three major contributions. The first feature (in local time at the observation point) is the sharp rise from the zero field to the peak value of the field. This is when the field components from the central part of the emitter reach the observation point. Later the contributions from the field-enhanced regions near the electrodes reach the observation point, resulting in two weaker maxima in the pulse shape. On the propagation axis these two contributions arrive simultaneously, but they can in principle be separated at off-axis positions. In the experimental data (Fig. 11) we resolve only the first peak and a second shoulder, delayed according to the off-axis position with respect to the first peak.

In Fig. 12 we show numerical integrations of the scalar and vectorial broadband diffraction integrals [Eqs. (2) and (8)]. In the same figure we show one set of experimental data to illustrate the agreement between the simulation and the experiments. In the simulation data the three main features of the pulse are visible. To obtain the simulated data in Fig. 12 we used $n = 6$ and $E_e/E_c = 2.5$ in Eq. (12).

In Fig. 13 we show a measurement of the THz pulse shape as the detector is moved from the near field toward the far field of the emitter. The pulse shape is shown for $z = 11 \text{ mm}$, $z = 20 \text{ mm}$, and $z = 81 \text{ mm}$. The transition into the far-field pulse shape is not complete in the series...
of pulses, but the general trend is clear. As the detector is moved further away from the emitter, the pulse shape develops from the typical multipeaked shape observed in the near field to the single-peaked THz pulse shape normally observed in the far field. Together with the experimental data we show results of the numerical integration of the scalar diffraction integral, Eq. (2).

5. CONCLUSIONS

We have described the propagation of ultrashort THz pulses emitted from a biased, large-aperture THz antenna. By measuring the frequency-dependent radiation pattern of an unfocused THz beam and a focused THz beam, we find that the THz beam essentially behaves like a fundamental Gaussian beam with a beam waist given by the spot size of the emitting laser pulse at the THz emitter. This fact is also true for other types of LA emitters, since the propagation characteristics are independent of the microscopic processes generating the THz beam. We have demonstrated that it is possible to model the full electrical field vector of the THz beam by a single diffraction integral and that we can obtain important information about the field distribution on the emitter surface by studying the THz pulse shape in the near field. The shape of the rising edge of the THz pulse is determined by the generation process in the emitter, whereas the shape of the trailing part is determined by the emitter geometry and propagation effects. We find that focusing of the THz beam leads to an enhancement of the peak THz field strength but that at the same time the normalized pulse momentum, the important quantity in THz field ionization experiments, is reduced.

ACKNOWLEDGMENTS

This work was supported by the Deutsche Forschungsgemeinschaft, project SFB 276, TP C14.

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REFERENCES


Fig. 12. Numerical determination of off-axis THz pulse shape at z = 16 mm. One experimental data set is shown to illustrate the agreement with experiment.

Fig. 13. On-axis THz pulse shape as a function of distance to the emitter.
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